A Topological Origin for Cosmic Birefringence: A Parameter-Free Prediction of $\beta_0 = 1/16\pi^2$

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Abstract

We derive a parameter-free value for the CMB polarization rotation angle, $\beta_0 = 1/(16\pi^2) = 0.36282976^\circ$, from first principles: a unit Pontryagin index on the natural $S^2 \times S^2$ boundary of a minimal geometric cell (a structured vacuum domain). The derivation is purely topological and independent of frequency, distance, or model parameters. We then show coherence with a second canonical relation, $\eta_c \equiv e^2/{e_{emax}}^2 = 8\pi\alpha$, obtained non-circularly by combining the same topology with an independently determined duality fill fraction from chronovibrational geometry (via a g-factor bridge). Finally, we present a metrologically closed definition of the Aether Fluctuation Potential aefp through the ledger identity $F_q = \text{mflx} \cdot \text{aefp}$, with two independent routes for aefp. We outline falsifiable tests with current and upcoming CMB polarization data and SI-traceable, QMU-reported metrology.

1 Introduction

Recent Planck analyses (Minami & Komatsu 2020; Diego-Palazuelos et al. 2022) hint at a uniform cosmic birefringence of ~ 0.30–0.35° at 2–3σ, motivating an exact prediction for the global duality angle. We propose and derive

$$\beta_0 = \frac{1}{16\pi^2} = 0.36282976^\circ$$

from a topological invariant on the boundary of a quantized, geometric medium—a *structured vacuum* endowed with two orthogonal curvature spheres (spatial flux and polarization duality). Within our full formulation this medium is historically termed the *Aether*, and the associated bookkeeping uses Quantum Measurement Units (QMU) [7, 8]. Those details are deferred to Materials & Methods so readers can assess the mathematics first.

2 Core Derivation: $\beta_0 = 1/(16\pi^2)$ from $S^2 \times S^2$

Geometric setup. Let $\mathcal{U}$ be a minimal structured-vacuum cell in 5-D ledger coordinates $(L_1, L_2, L_3, F_{\text{fwd}}, F_{\text{torq}})$. Its boundary factorizes as $\partial\mathcal{U} \simeq S^2_L \times S^2_F$, carrying a $U(1)$ connection A and curvature two-form $F = dA$, with $F = F_L + F_F$ supported on the respective 2-spheres.

Quantization (first Chern classes). By construction,

$$\int_{S^2_L} F_L = 4\pi, \qquad \int_{S^2_F} F_F = 4\pi,$$

so each sphere has unit first Chern number.

Pontryagin normalization. On $\partial\mathcal{U}$, write $F = F_L + F_F$. The diagonal pieces $F_L \wedge F_L$ and $F_F \wedge F_F$ vanish by support on their respective 2-spheres, while 2-forms commute in 4D so the cross-terms add. We choose the standard normalization so that

$$\int_{\partial\mathcal{U}} F \wedge F = \Big(\int_{S^2_L} F_L\Big)\Big(\int_{S^2_F} F_F\Big) = (4\pi)(4\pi) = 16\pi^2,$$

i.e. any overall factor from the commuting-forms convention is absorbed into $16\pi^2$. The normalized Pontryagin index is therefore $Q_{\mathcal{U}} = \frac{1}{16\pi^2}\int_{\partial\mathcal{U}} F \wedge F = 1$.

From topology to a duality angle. Using the axion-like term $\Delta\mathcal{L} = \kappa\, F \wedge F$ (see the margin note in Sec. 3), the action shift per minimal cell is $\Delta S_{\mathcal{U}} = \kappa\, 16\pi^2 = \kappa\, 16\pi^2$. Homogeneous post-recombination κ produces a global duality tilt $\beta_0 = \kappa_0$ once mapped to the EFT conventions below. The parameter-free choice consistent with $Q_{\mathcal{U}} = 1$ is $\kappa_0 = 1/16\pi^2$, thus

$$\beta_0 = \frac{1}{16\pi^2} = \frac{1}{16\pi^2}.$$

3 EFT Translation and Polarization Rotation

Start from $S = \int d^4x\big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}\theta(\eta)F_{\mu\nu}\tilde{F}^{\mu\nu}\big]$. Varying yields $\nabla_\mu F^{\mu\nu} + (\nabla_\mu\theta)\tilde{F}^{\mu\nu} = 0$, $\nabla_\mu\tilde{F}^{\mu\nu} = 0$. For plane waves, $A_\pm \propto e^{-ik\eta}e^{\pm i\theta/2}$, and a linear polarization rotates by $\beta = \frac{1}{2}[\theta(\eta_0) - \theta(\eta_*)]$. Using the identity $F_{\mu\nu}\tilde{F}^{\mu\nu}\, d^4x = 2\, F\wedge F$, the topological term $\Delta\mathcal{L} = \kappa\, F\wedge F$ corresponds to $\frac{1}{4}\theta F\tilde{F}$ with

$$\boxed{\theta = 2\kappa}.$$

For a homogeneous late-time $\kappa = \kappa_0$, $\beta_0 = \frac{1}{2}\theta_0 = \kappa_0$. From Sec. 2, $\kappa_0 = 1/16\pi^2 \Rightarrow \beta_0 = 1/16\pi^2$.

Identity used: $F_{\mu\nu}\tilde{F}^{\mu\nu}\, d^4x = 2\, F\wedge F$.

4 Coherence: $\eta_c = 8\pi\alpha$ without circularity

Product rule from topology. For submaximal excitations, $F = \eta_c\,\omega_L + \xi\,\omega_F$, where $\int_{S_L^2}\omega_L = \int_{S_F^2}\omega_F = 4\pi$, $\eta_c \equiv e^2/{e_{emax}}^2$ is the spatial-flux fill fraction, and ξ is a duality fill fraction. Then

$$\int_{\partial\mathcal{U}} F\wedge F = \eta_c\,\xi\,(4\pi)^2 \quad\Rightarrow\quad \eta_c\,\xi = 1,$$

using $Q_{\mathcal{U}} = 1$ under the same normalization convention.

Independent evaluation of ξ. The chronovibrational geometry that reproduces the g-factor series via an APM–SM bridge fixes the net duality fill to $\xi = 1/(8\pi\alpha)$ (see [9] for construction and tests). Combining with $\eta_c\,\xi = 1$ yields

$$\eta_c = 8\pi\alpha, \qquad {e_{emax}}^2 = \frac{e^2}{8\pi\alpha},$$

with no circular dependence on the target relation.

5 Experimental Tests

5.1 CMB Polarization

Target: $\beta_0 = 0.36282976^\circ$, uniform and achromatic. Pipeline: preregistered β_0; blind EB/TB analysis with self-calibration; multi-frequency Faraday rejection; foreground-masked nulls.

5.2 Independent Charge-Ratio Metrology

Quantum Hall plateau + Josephson chain to extract ${e_{emax}}^2$; falsify if η_c deviates from $8\pi\alpha$ by $> 5\sigma$.

5.3 Cross-Checks with Astrophysical Polarization

Compare CMB β_0 to extragalactic radio/optical polarization; anisotropy falsifies the uniform prediction.

6 Materials & Methods: Framework and QMU Ledger

6.1 QMU Primer (minimal ledger)

We use $(m_{\mathrm{e}}, \lambda_{\mathrm{C}}, F_q, e^2, {e_{emax}}^2)$ as the minimal set; derived mflx and base-candidate aefp close the photon–charge ledger [7, 8].

- **Chronovibration:** $F_q = \mathrm{mflx}\cdot\mathrm{aefp} = c/\lambda_{\mathrm{C}}$ (ledger identity).
- **Charge ratio:** $\eta_c = e^2/{e_{emax}}^2 = 8\pi\alpha$ (derived in Sec. 4).

6.2 Operational Definition and Metrological Tests of aefp

Promote aefp to base-unit status via

$$F_q = \text{mflx} \cdot \text{aefp}, \qquad \text{aefp} = \frac{F_q}{\text{mflx}}, \qquad \text{aefp} = \frac{{e_{emax}}^2}{m_\text{e}\,{\lambda_\text{C}}^2},$$

with closure residual $\Delta_\text{aefp} = (\text{aefp}^{(A)} - \text{aefp}^{(B)})/\text{aefp}^{(B)}$ and preregistered tolerance ϵ_aefp. Species-scaling null: $\frac{\text{aefp}_x}{\text{aefp}_\text{e}} = \frac{m_\text{e}}{m_x}\left(\frac{\lambda_\text{C}}{\lambda_{C,x}}\right)^2$.

Canonical Invariants Summary

Invariant	**Definition (QMU)**	**Pure QMU Expression**	**Value / SI Cross-Check**
CMB Rotation Angle β_0	$\dfrac{1}{16\pi^2}$	Canonical holonomy tilt of the structured vacuum geometry	$0.006332574\,\text{rad} = 0.36282976^\circ$
Charge-Ratio η_c	$\dfrac{e^2}{{e_{emax}}^2}$	With ${e_{emax}}^2 = e^2/(8\pi\alpha)$, so $\eta_c = 8\pi\alpha$	$0.1834024736 \pm 1.5 \times 10^{-11}$
Chronovibration Frequency F_q	$\dfrac{c}{\lambda_\text{C}}$	$F_q = \text{mflx} \cdot \text{aefp}$	$1.23559 \times 10^{20}\,\text{s}^{-1}$ (electron case)

7 Uncertainty Propagation

- $\beta_0 = 0.36282976^\circ$, falsified if $|\beta_\text{obs} - \beta_0| > 5\sigma$.
- $\eta_c = 0.1834024736 \pm 1.5 \times 10^{-11}$.
- $F_q = \text{mflx} \cdot \text{aefp} = c/\lambda_\text{C}$ (ledger identity; SI cross-checks in the appendix).

8 Systematics Playbook

- Angle calibration: astrophysical calibrators and sky-rotation nulls.
- Faraday rotation: verify absence of λ^2 dependence.
- Foregrounds: vary masks; cross-check Planck/SO/LiteBIRD.
- Instrumental leakage: sky-rotation and deck-angle null tests.

9 Comparative Context

Axion/Chern–Simons birefringence models introduce free parameters. Here the duality angle is fixed by topology: $\beta_0 = 1/16\pi^2$. Likewise, $\eta_c = 8\pi\alpha$ follows from AU topology and an

independently obtained duality fill fraction. Both results are dispersionless and parameter-free.

10 Kill-Shot Matrix

Prediction	**Falsification Criterion**
$\beta_0 = 0.3628^\circ$	$\lvert\beta_{\rm obs} - 0.3628^\circ\rvert > 5\sigma$
Dispersionless	Detection of frequency dependence
Uniform sky	Significant anisotropy in EB/TB spectra
$\eta_c = 0.1834$	$> 5\sigma$ deviation in QMU-reported, SI-traceable metrology of ${e_{emax}}^2$
$F_q = c/\lambda_{\rm C}$	Discrepancy in Compton-frequency determinations across species (QMU ledger)
aefp base-unit status	Closure $\lvert\Delta_{\rm aefp}\rvert < \epsilon_{\rm aefp}$ *and* species-ratio concordance; falsified if violation exceeds $\epsilon_{\rm aefp}$

11 Discussion

Leading with the topological derivation makes the work easy to audit: the $S^2 \times S^2$ construction fixes β_0, and the same boundary geometry, combined with a chronovibrational fill fraction from the g-factor bridge [9], fixes η_c without circularity. Beyond birefringence, this geometric program interfaces naturally with other puzzles: the electron/muon g-factors (and the muon $g-2$ tension) arise from the same chronovibrational projection, suggesting a common geometric origin for multiple anomalies.

12 Conclusion

A single, unit Pontryagin index on the boundary of a minimal structured-vacuum cell yields $\beta_0 = 1/(16\pi^2)$, and, together with an independently determined duality fill fraction, fixes $\eta_c = 8\pi\alpha$. We operationalize aefp through a metrological closure, $F_q = \text{mflx} \cdot \text{aefp}$, with two independent routes and species-scaling nulls. Upcoming CMB polarization data and QMU-reported, SI-traceable metrology can decisively confirm or falsify this geometry-first account of birefringence and charge structure.

A SI Cross-Check Box

- $\alpha^{-1} = 137.0359992(\ldots) \Rightarrow \eta_c = 8\pi\alpha = 0.1834024736$
- $c/\lambda_{\rm C} = 1.23559 \times 10^{20}\,{\rm s}^{-1}$ (electron case)
- Angles in degrees for convenience; ledger uses radians.

B Systematics Stress-Tests

Detailed protocols for foreground masking, angle calibration, Faraday nulls, and leakage checks.

C QMU Materials & Methods Cheat Sheet

Purpose. This one-page reference lets reviewers reproduce the ledger calculations and metrology without reading the full framework.

Minimal Ledger (Base + Derived)

- **Base:** $m_{\mathrm{e}},\ \lambda_{\mathrm{C}},\ F_q,\ e^2,\ {e_{emax}}^2$.
- **Derived:** mflx, aefp with

$$F_q = \mathrm{mflx} \cdot \mathrm{aefp}, \qquad \mathrm{mflx} = \frac{m_{\mathrm{e}}\, F_q}{{e_{emax}}^2}, \qquad \mathrm{aefp} = \frac{{e_{emax}}^2}{m_{\mathrm{e}}\, {\lambda_{\mathrm{C}}}^2}.$$

- **Canonical ratios:**

$$\beta_0 = \frac{1}{16\pi^2}, \qquad \eta_c \equiv \frac{e^2}{{e_{emax}}^2} = 8\pi\alpha.$$

Quick Reference Table

Quantity	Definition (QMU)	Operational route (ledger)
CMB duality angle β_0	$\dfrac{1}{16\pi^2}$	EB/TB self-cal; dispersionless, sky-uniform test
Charge ratio η_c	$\dfrac{e^2}{{e_{emax}}^2} = 8\pi\alpha$	Extract ${e_{emax}}^2$ via quantum-metrology chain; check against $8\pi\alpha$
Chronovibration F_q	$\dfrac{c}{\lambda_{\mathrm{C}}}$	Ledger identity: verify $F_q = \mathrm{mflx} \cdot \mathrm{aefp}$
Flux unit mflx	$\dfrac{m_{\mathrm{e}}\, F_q}{{e_{emax}}^2}$	From plateau indices/Josephson phase steps
Aether fluctuation aefp	$\dfrac{{e_{emax}}^2}{m_{\mathrm{e}}\, {\lambda_{\mathrm{C}}}^2}$ and $\dfrac{F_q}{\mathrm{mflx}}$	Dual route closure: $\mathrm{aefp}^{(A)} = F_q/\mathrm{mflx}$, $\mathrm{aefp}^{(B)} = {e_{emax}}^2/(m_{\mathrm{e}}{\lambda_{\mathrm{C}}}^2)$

Metrological Closure for aefp

$$\Delta_{\text{aefp}} \;\equiv\; \frac{\text{aefp}^{(A)} - \text{aefp}^{(B)}}{\text{aefp}^{(B)}}, \qquad \text{require } |\Delta_{\text{aefp}}| < \epsilon_{\text{aefp}} \;\text{ and species-ratio concordance.}$$

Species scaling (any massive species x):

$$\frac{\text{aefp}_x}{\text{aefp}_{\text{e}}} = \frac{m_{\text{e}}}{m_x}\left(\frac{\lambda_{\text{C}}}{\lambda_{C,x}}\right)^2.$$

CMB EB/TB Extraction (Small-Angle Ledger Form)

Rotation by β mixes E/B as

$$E' = E\cos 2\beta - B\sin 2\beta, \qquad B' = E\sin 2\beta + B\cos 2\beta.$$

Small-angle estimators (per multipole band):

$$C_\ell^{EB} \;\simeq\; 2\beta\, C_\ell^{EE}, \qquad C_\ell^{TB} \;\simeq\; 2\beta\, C_\ell^{TE}.$$

Practical estimator (band-averaged):

$$\hat{\beta} \;\approx\; \frac{1}{2}\frac{C^{EB}}{C^{EE}} \quad \text{and} \quad \hat{\beta} \;\approx\; \frac{1}{2}\frac{C^{TB}}{C^{TE}},$$

with consistency required across frequencies and masks; any λ^2 trend falsifies dispersionlessness.

Quantum-Metrology Recipe (Ledger-Only Reporting)

1. **Flux unit** mflx**:** determine from quantized plateau index and Josephson phase steps; report mflx directly in ledger units.
2. **Chronovibration** F_q**:** obtain from Compton-linked tick; report F_q.
3. **Route A:** $\text{aefp}^{(A)} = F_q/\text{mflx}$.
4. **Route B:** measure ${e_{emax}}^2$ via charge-ratio pathway; compute $\text{aefp}^{(B)} = {e_{emax}}^2/(m_{\text{e}}{\lambda_{\text{C}}}^2)$.
5. **Closure:** evaluate Δ_{aefp}; apply preregistered ϵ_{aefp}.

Pre-Registration (Hash-Lock Sketch)

Include a plain-text blob (ledger symbols only), for example:

```
beta0 := 1/(16*pi^2)
eta_c := 8*pi*alpha
Fq := mflx * aefp
aefpA := Fq / mflx
aefpB := e_emax^2 / (m_e * lambda_C^2)
tolerance_epsilon_aefp := ...
masks := { ... }, bands := { ... }, EB/TB estimator := ...
```

Commit the blob's cryptographic hash alongside analysis start time; publish both.

Common Pitfalls (and Fixes)

- **Angle conventions:** ledger uses radians; keep 2β factors consistent in EB/TB mixing.
- **Self-cal leakage:** instrument angle self-cal can absorb uniform β; use external calibrators and sky-rotation nulls.
- **Foregrounds:** verify invariance under mask changes; dust residuals can bias C^{EB}.
- **Dispersion:** any frequency dependence ($\propto \lambda^2$) indicates Faraday, not duality holonomy.
- **Circularity:** when checking η_c, never infer ${e_{emax}}^2$ from α; determine ${e_{emax}}^2$ operationally, then compare to $8\pi\alpha$.

Notation Map (for Cross-Reading)

- Structured-vacuum cell: $\mathcal{U}$, boundary $\partial\mathcal{U} \simeq S^2_L \times S^2_F$.
- Curvature split: $F = F_L + F_F$, with $\int_{S^2_L} F_L = \int_{S^2_F} F_F = 4\pi$.
- Holonomy index: $Q_{\mathcal{U}} = \frac{1}{16\pi^2}\int_{\partial\mathcal{U}} F \wedge F = 1$.
- EFT bridge: $\Delta\mathcal{L} = \kappa\, F \wedge F$, $\beta_0 = \kappa_0$ with $\theta = 2\kappa$.

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