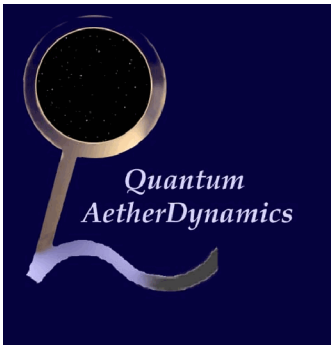

Quantum AetherDynamics Institute (QADI)

Operator Package 3: Coherence-Filtered Spectral Selection on Finite Log-Scale Windows in APM/QMU

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Abstract

Operator Packages 1–2 establish (i) a log-scale spectral backbone and (ii) a physically realizable discretization on a finite log-scale window with holonomy-controlled boundary conditions, together with a compact xi-kernel filter W_I [Thomson III, 2026a,b]. Operator Package 3 introduces the first *coherence-filtered selection* mechanism: we combine an APM discretized Hamiltonian H_{APM} with the compact filter W_I through spectral projectors to form a finite-rank self-adjoint selection operator

$$\mathcal{S}_{E_c} := \Pi_{E_c} W_I \Pi_{E_c}, \qquad \Pi_{E_c} = \mathbf{1}_{(-\infty, E_c]}(H_{\mathrm{APM}}).$$

This object makes the philosophy operational: spectra are not merely mapped, they are *filtered* by coherence constraints (self-adjointness, domain regularity, finite resolution). We prove the basic operator-theoretic properties of $\mathcal{S}_{E_c}$ (self-adjointness, finite rank, variational characterizations, stability bounds), provide explicit matrix representations in both the scale-harmonic basis and the Hamiltonian eigenbasis, and record a calibration example ($V_{\mathrm{APM}} \equiv 0$) where the Fejér smoothing induced by finite windows becomes exact. In this coherence-baseline limit, the Hamiltonian eigenbasis coincides with the Package 2 scale-harmonic basis, and the matrix elements of $\mathcal{S}_{E_c}$ recover the same Fejér-smoothed diagonal sampling derived there. We conclude with an APM instantiation template for $V_{\mathrm{APM}}(q)$ and a computational protocol intended to support future mode-prediction papers.

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1 Purpose and scope

1.1 What Package 3 adds

Operator Package 1 provides the infinite-line analytic backbone (P and $W = \Xi(P)$), while Operator Package 2 provides a finite-window realization with boundary holonomy and a compact xi-kernel filter W_I (including Fejér smoothing as the unavoidable finite-resolution effect) [Thomson III, 2026a,b]. Package 3 introduces the *selection principle* that turns these ingredients into a computable prediction engine:

- choose a self-adjoint discretized APM Hamiltonian H_{APM} on a finite log-scale window;
- choose a compact self-adjoint filter W_I on the same window (xi-kernel benchmark);
- form a finite-rank self-adjoint selection operator $\mathcal{S}_{E_c} = \Pi_{E_c} W_I \Pi_{E_c}$ and read out its leading spectral features as coherence-surviving modes.

The key deliverable is a standardized chain of definitions and theorems that can be cited verbatim by subsequent QADI/APM papers.

1.2 Relationship to engineered "Riemann-zero Hamiltonians"

Recent mathematical-physics constructions engineer Hamiltonians whose assigned energies reproduce functions of the nontrivial zeta zeros, while noting that such mappings need not resolve Hilbert–Pólya if the corresponding states fail to live in a standard L^2 Hilbert space [Suo, 2025]. Package 3 takes

the complementary stance: start from a coherence-valid Hilbert space and self-adjoint operators on a finite window; then characterize what spectral content survives the compact filter and the finite-resolution constraints.

2 Finite log-scale window and coherence domains (Package 2 recap)

2.1 QMU-normalized log-scale coordinate

We use the APM/QMU log-scale coordinate

$$q := \log\left(\frac{\ell}{\lambda_C}\right), \qquad \ell(q) = \lambda_C e^q, \tag{1}$$

so translations in q represent multiplicative rescalings of the length scale ℓ in Compton units.

Define a finite log-scale window

$$I := [q_{\min}, q_{\max}], \qquad L_q := q_{\max} - q_{\min} > 0, \tag{2}$$

and the Hilbert space

$$\mathcal{H}_I := L^2(I,\, \mathrm{d}q), \qquad \langle f, g\rangle_I := \int_{q_{\min}}^{q_{\max}} f(q)\overline{g(q)}\, \mathrm{d}q.$$

2.2 Holonomy-controlled quasi-periodic boundary conditions

Definition 2.1 (Holonomy boundary parameter)**.** Fix a phase $\vartheta \in \mathbb{R}$ (interpretable as APM holonomy/closure phase). Define the quasi-periodic boundary conditions

$$f(q_{\max}) = e^{\mathrm{i}\vartheta} f(q_{\min}), \qquad f'(q_{\max}) = e^{\mathrm{i}\vartheta} f'(q_{\min}). \tag{3}$$

Remark 2.2. The pair (3) is the natural closure class ensuring symmetry under integration by parts for both first- and second-order differential operators. In APM language, ϑ is the "holonomy dial" controlling discrete scale-harmonics.

2.3 The scale generator and its discrete spectrum

Definition 2.3 (Self-adjoint scale generator on the window)**.** Define

$$P_\vartheta := -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}q}$$

with domain

$$\mathrm{Dom}(P_\vartheta) := \{f \in H^1(I) : f(q_{\max}) = e^{\mathrm{i}\vartheta} f(q_{\min})\}.$$

Theorem 2.4 (Self-adjointness and eigenbasis of P_ϑ)**.** *P_ϑ is self-adjoint on $\mathcal{H}_I$ and has a complete orthonormal eigenbasis*

$$e_n(q) := \frac{1}{\sqrt{L_q}} \exp\Big(\mathrm{i}t_n(q - q_{\min})\Big), \qquad n \in \mathbb{Z}, \tag{4}$$

with eigenvalues

$$t_n := \frac{2\pi n + \vartheta}{L_q}, \qquad P_\vartheta e_n = t_n e_n. \tag{5}$$

Proof. A standard integration-by-parts computation shows P_ϑ is symmetric on $\mathrm{Dom}(P_\vartheta)$, and the quasi-periodic boundary condition eliminates the boundary form. Completeness and orthonormality follow from the standard Fourier basis on a finite interval with a phase twist [Reed and Simon, 1980]. □

2.4 Ledger normalization via APM propagation constant

Recall Ledger One closure in QMU form:

$$A_u \cdot \mathrm{curl} = {F_q}^2 {\lambda_C}^2. \tag{6}$$

Define the QMU propagation constant

$$\kappa_0 := F_q \lambda_C = F_q \, \lambda_C. \tag{7}$$

Introduce the ledger-normalized scale generator

$$P_{\mathrm{L}} := \kappa_0 \, P_\vartheta. \tag{8}$$

This is the canonical APM/QMU normalization for upgrading the dimensionless log-scale spectrum $\{t_n\}$ to ledger-scaled spectral quantities.

3 The discretized APM Hamiltonian and the xi-kernel filter

3.1 Hamiltonian template

Definition 3.1 (Discretized APM Hamiltonian)**.** Let $V_{\mathrm{APM}} : I \to \mathbb{R}$ be bounded and measurable. Define the Sturm–Liouville operator

$$H_{\mathrm{APM}} := -\frac{\mathrm{d}^2}{\mathrm{d}q^2} + V_{\mathrm{APM}}(q) \tag{9}$$

with domain

$$\mathrm{Dom}(H_{\mathrm{APM}}) := \{f \in H^2(I) : f \text{ satisfies } (3)\}.$$

Theorem 3.2 (Self-adjointness and discrete spectrum)**.** *H_{APM} is self-adjoint on $\mathcal{H}_I$ and has compact resolvent; hence $\sigma(H_{\mathrm{APM}})$ is purely discrete with eigenvalues accumulating only at $+\infty$.*

Proof. On a bounded interval, a real bounded potential is relatively bounded with respect to $-\mathrm{d}^2/\mathrm{d}q^2$. The quasi-periodic boundary conditions define a self-adjoint realization of the second derivative; adding V_{APM} preserves self-adjointness. Compactness of the resolvent follows from standard compact-embedding/elliptic arguments on bounded intervals; see, e.g., Reed and Simon [1980]. □

3.2 The finite-window xi-kernel filter

Let Φ be the real, even, rapidly decaying xi-kernel from Operator Package 1 (theta-built). We use the Fourier-pair convention

$$\Xi(t) = \int_{-\infty}^{\infty} \Phi(v)\, e^{-\mathrm{i}tv}\, \mathrm{d}v, \tag{10}$$

with the understanding that the sign choice is immaterial here because Φ is even and Ξ is even.

Definition 3.3 (Finite-window xi-filter)**.** Define $W_I : \mathcal{H}_I \to \mathcal{H}_I$ by

$$(W_I f)(q) := \int_{q_{\min}}^{q_{\max}} \Phi(q-u)\, f(u)\, \mathrm{d}u, \qquad q \in I. \tag{11}$$

Theorem 3.4 (Compact self-adjointness of W_I)**.** *W_I is bounded, self-adjoint, and compact on $\mathcal{H}_I$. In fact, W_I is Hilbert–Schmidt with kernel $\Phi(q-u)$ restricted to $I \times I$.*

Proof. Continuity and square-integrability of the restricted kernel on a bounded square imply the operator is Hilbert–Schmidt, hence compact and bounded. Since Φ is real and even, the kernel is symmetric: $\Phi(q-u) = \overline{\Phi(u-q)}$, giving self-adjointness by a Fubini argument [Reed and Simon, 1980]. □

4 Operator Package 3: the coherence-filtered selection operator

4.1 Spectral projectors and finite-rank selection

Definition 4.1 (Energy-cutoff spectral projector)**.** For $E_c \in \mathbb{R}$, define the spectral projector

$$\Pi_{E_c} := \mathbf{1}_{(-\infty, E_c]}(H_{\mathrm{APM}}), \tag{12}$$

i.e. orthogonal projection onto the span of eigenfunctions of H_{APM} with eigenvalues $\lambda \le E_c$ [Reed and Simon, 1980].

Definition 4.2 (Selection operator)**.** Define the coherence-filtered selection operator

$$\mathcal{S}_{E_c} := \Pi_{E_c}\, W_I\, \Pi_{E_c}. \tag{13}$$

Theorem 4.3 (Basic properties of $\mathcal{S}_{E_c}$)**.** *For any $E_c \in \mathbb{R}$, the operator $\mathcal{S}_{E_c}$ is bounded, self-adjoint, and finite rank. Moreover,*

$$\|\mathcal{S}_{E_c}\| \le \|W_I\|, \tag{14}$$

and $\sigma(\mathcal{S}_{E_c})$ is a finite set of real eigenvalues (counting multiplicity).

Proof. Π_{E_c} is an orthogonal projection (hence bounded and self-adjoint). W_I is bounded and self-adjoint by Theorem 3.4. Therefore $\mathcal{S}_{E_c}$ is self-adjoint. Since H_{APM} has discrete spectrum with eigenvalues $\lambda_k \to +\infty$, there are only finitely many $\lambda_k \le E_c$, so $\mathrm{Ran}(\Pi_{E_c})$ is finite-dimensional and $\mathcal{S}_{E_c}$ is finite rank. The norm bound follows from $\|\Pi_{E_c}\| = 1$ and submultiplicativity. □

4.2 Variational characterizations ("coherence survival" principles)

Theorem 4.4 (Top-mode variational principle)**.** *Let $\Lambda_{\max}(E_c)$ be the largest eigenvalue of $\mathcal{S}_{E_c}$. Then*

$$\Lambda_{\max}(E_c) = \max_{\substack{f \in \mathrm{Ran}(\Pi_{E_c}) \\ \|f\|_I = 1}} \langle f, W_I f \rangle_I. \tag{15}$$

Similarly, the smallest eigenvalue is the corresponding minimum [Reed and Simon, 1980].

Proof. $\mathcal{S}_{E_c}$ is self-adjoint on the finite-dimensional space $\mathrm{Ran}(\Pi_{E_c})$, so the Rayleigh–Ritz principle applies. For $f \in \mathrm{Ran}(\Pi_{E_c})$, $\Pi_{E_c} f = f$ and Π_{E_c} is orthogonal, hence

$$\langle f, \mathcal{S}_{E_c} f \rangle_I = \langle f, \Pi_{E_c} W_I \Pi_{E_c} f \rangle_I = \langle f, W_I f \rangle_I.$$

Maximizing/minimizing over unit vectors in $\mathrm{Ran}(\Pi_{E_c})$ yields the extremal eigenvalues. □

Definition 4.5 (Two scalar selection functionals)**.** Define

$$\mathcal{J}(E_c) := \mathrm{Tr}(\mathcal{S}_{E_c}) = \sum_{\lambda_k \le E_c} \langle \psi_k, W_I \psi_k \rangle_I, \tag{16}$$

$$\Lambda_{\max}(E_c) := \max \sigma(\mathcal{S}_{E_c}). \tag{17}$$

Remark 4.6. In APM terms, $\mathcal{J}(E_c)$ measures total filter acceptance within the coherence-admissible energy band; $\Lambda_{\max}(E_c)$ identifies the single most filter-aligned coherent mode. Both are basis-invariant and computable.

4.3 A stability bound that is always true

Theorem 4.7 (Lipschitz bound in the filter)**.** *If W_I and $\widetilde{W}_I$ are two bounded self-adjoint filters on $\mathcal{H}_I$, then for fixed H_{APM} and E_c,*

$$\|\Pi_{E_c} W_I \Pi_{E_c} - \Pi_{E_c} \widetilde{W}_I \Pi_{E_c}\| \le \|W_I - \widetilde{W}_I\|.$$

Consequently,

$$|\Lambda_{\max}(E_c) - \widetilde{\Lambda}_{\max}(E_c)| \le \|W_I - \widetilde{W}_I\|.$$

Proof. Use $\|\Pi_{E_c}\| = 1$ and submultiplicativity to obtain the operator-norm bound. On the finite-dimensional space $\mathrm{Ran}(\Pi_{E_c})$, the max-eigenvalue perturbation bound gives $|\lambda_{\max}(A) - \lambda_{\max}(B)| \le \|A - B\|$ for self-adjoint matrices A, B. □

Remark 4.8 (Optional strengthened stability in H_{APM})**.** Stability with respect to perturbations in V_{APM} can be made quantitative when an eigenvalue gap separates $\mathrm{Ran}(\Pi_{E_c})$ from the complement; then standard invariant-subspace perturbation bounds control $\|\Pi_{E_c} - \widetilde{\Pi}_{E_c}\|$. This refinement is useful in numerical scans of V_{APM}, but is not required for the definition of Package 3.

5 Matrix representations in the two natural bases

5.1 Hamiltonian eigenbasis

Let $\{(\lambda_k, \psi_k)\}_{k\geq 1}$ be an orthonormal eigenpair sequence for H_{APM}, ordered nondecreasingly. Let $K(E_c) := \#\{k : \lambda_k \leq E_c\}$ and define the finite-dimensional subspace

$$\mathcal{H}_{E_c} := \mathrm{Ran}(\Pi_{E_c}) = \mathrm{span}\{\psi_1, \ldots, \psi_{K(E_c)}\}.$$

In this basis, $\mathcal{S}_{E_c}$ is represented by the $K(E_c) \times K(E_c)$ Hermitian matrix

$$(\mathsf{S}_{E_c})_{jk} := \langle \psi_j, W_I \psi_k \rangle_I, \qquad 1 \leq j, k \leq K(E_c). \tag{18}$$

Computing S_{E_c} reduces to evaluating kernel integrals on $I \times I$.

5.2 Scale-harmonic basis and overlap factors

Let $\{e_n\}_{n\in\mathbb{Z}}$ be the P_ϑ eigenbasis from Theorem 2.4. Define overlap coefficients

$$U_{kn} := \langle \psi_k, e_n \rangle_I. \tag{19}$$

Then, formally (and exactly when the e_n expansion is truncated to a finite index set),

$$\langle \psi_j, W_I \psi_k \rangle_I = \sum_{m,n} \overline{U_{jm}}\, \langle e_m, W_I e_n \rangle_I\, U_{kn}. \tag{20}$$

This identity makes transparent how W_I is "almost diagonal" in the scale-harmonic basis while H_{APM} is diagonal in the ψ_k basis.

6 Calibration case: $V_{\mathrm{APM}} \equiv 0$ and exact Fejér smoothing

6.1 Connection to Package 2 (coherence baseline)

In the limit $V_{\mathrm{APM}} \equiv 0$, the Hamiltonian eigenbasis coincides with the Package 2 scale-harmonic basis $\{e_n\}$. Consequently, the matrix elements of the Package 3 selection operator recover the exact Fejér-smoothed diagonal form already derived in the finite-window analysis of Package 2 [Thomson III, 2026b], now interpreted as the baseline "coherence + resolution" response against which nontrivial potentials V_{APM} can be compared.

6.2 Free-window Hamiltonian and shared eigenbasis

Definition 6.1 (Free-window Hamiltonian)**.** Set $V_{\mathrm{APM}}(q) \equiv 0$ and define

$$H_0 := -\frac{\mathrm{d}^2}{\mathrm{d}q^2}$$

with the quasi-periodic boundary conditions (3).

Lemma 6.2 (Shared eigenbasis)**.** *H_0 diagonalizes in the same basis $\{e_n\}$ as P_ϑ, with*

$$H_0 e_n = t_n^2\, e_n,$$

where t_n is given by (5).

Proof. Differentiate (4) twice. The quasi-periodic derivative condition is satisfied automatically because e_n is exponential. □

6.3 Exact diagonal formula and Fejér kernel

Theorem 6.3 (Exact windowed diagonal and Fejér smoothing)**.** *Define* $\mathrm{sinc}(y) := \frac{\sin y}{y}$ *(with* $\mathrm{sinc}(0) := 1$*). For each $n \in \mathbb{Z}$,*

$$\langle e_n, W_I e_n\rangle_I = \frac{1}{L_q}\int_{-L_q}^{L_q} (L_q - |v|)\,\Phi(v)\, e^{-\mathrm{i}t_n v}\,\mathrm{d}v. \tag{21}$$

Equivalently, there exists a nonnegative even Fejér kernel F_{L_q} on $\mathbb{R}$ such that

$$\langle e_n, W_I e_n\rangle_I = \int_{-\infty}^{\infty} \Xi(\tau)\, F_{L_q}(t_n - \tau)\,\mathrm{d}\tau, \tag{22}$$

where

$$F_{L_q}(x) := \frac{1}{2\pi}\, L_q\, \mathrm{sinc}^2\Big(\frac{L_q x}{2}\Big), \qquad \int_{-\infty}^{\infty} F_{L_q}(x)\,\mathrm{d}x = 1. \tag{23}$$

Proof. Starting from the definition,

$$\langle e_n, W_I e_n\rangle_I = \frac{1}{L_q}\int_I\int_I \Phi(q-u)\, e^{-\mathrm{i}t_n(q-u)}\,\mathrm{d}u\,\mathrm{d}q.$$

Set $v = q - u$; for fixed $v \in [-L_q, L_q]$, the set of $(q,u) \in I \times I$ with $q - u = v$ has measure $L_q - |v|$. This yields (21).

For the convolution form, use the inverse Fourier representation corresponding to (10):

$$\Phi(v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Xi(\tau)\, e^{\mathrm{i}\tau v}\,\mathrm{d}\tau,$$

insert into (21), and interchange integrals. The remaining v-integral is the Fourier transform of the triangular window

$$A_{L_q}(v) := (L_q - |v|)\mathbf{1}_{|v|\le L_q},$$

namely

$$\int_{-L_q}^{L_q} (L_q - |v|)\, e^{-\mathrm{i}xv}\,\mathrm{d}v = L_q^2\, \mathrm{sinc}^2\Big(\frac{xL_q}{2}\Big),$$

a standard harmonic-analysis identity [Stein and Shakarchi, 2003]. Dividing by L_q and 2π yields (22) with kernel (23). The normalization $\int F_{L_q} = 1$ follows from $\int_{-\infty}^{\infty} \mathrm{sinc}^2(y)\,\mathrm{d}y = \pi$ under $y = L_q x/2$ [Stein and Shakarchi, 2003]. □

Remark 6.4 (Interpretation)**.** Finite APM systems do not access the pointwise multiplier $\Xi(t)$; they access a Fejér-averaged version whose resolution width scales as $\Delta t \sim L_q^{-1}$. This is the quantitative bridge between "measurement has finite scale" and the log-scale spectral picture.

6.4 Off-diagonal mode mixing (window-edge effect)

Remark 6.5 (Mode mixing locality)**.** On the infinite line, convolution diagonalizes exactly in Fourier modes. On a finite window, edge truncation induces off-diagonal couplings $\langle e_m, W_I e_n\rangle_I$ for $m \neq n$. The same triangular-window Fourier factor that produces the Fejér kernel controls the baseline suppression versus the scale-frequency separation $|t_m - t_n|$ (a sinc^2 envelope from the window), so mode mixing is localized in scale-frequency [Stein and Shakarchi, 2003].

7 APM instantiation template for $V_{\mathrm{APM}}(q)$

7.1 Ledger-normalized Hamiltonian

Using $\kappa_0 = F_q\,\lambda_C$, define the ledger-normalized Hamiltonian

$$H_{\mathrm{L}} := \kappa_0^2\, H_{\mathrm{APM}} = -\kappa_0^2\,\frac{\mathrm{d}^2}{\mathrm{d}q^2} + \kappa_0^2\, V_{\mathrm{APM}}(q). \tag{24}$$

This is the natural QMU normalization for comparing spectra across systems: q is dimensionless, and κ_0 is the Ledger One propagation scale.

7.2 A baseline APM potential family

Package 3 does not require selecting a specific V_{APM}, but the following family is a convenient APM-ready placeholder:

$$V_{\mathrm{APM}}(q;\eta) := V_0 + \eta\, U(q), \tag{25}$$

where U is a real bounded profile encoding packing/closure structure (e.g., a localized "packing defect" bump, or a periodic torsion profile over the window). The parameter η acts as an APM constitutive dial, analogous to a stiffness or coupling strength in ledger-normalized units.

Remark 7.1. Subsequent application papers can specify $U(q)$ from a concrete Aether-unit packing geometry or from an empirically calibrated constitutive profile; Package 3 remains valid for any bounded real U.

8 Computational protocol (ready-to-implement)

1. **Choose window and holonomy:** fix $(q_{\min}, q_{\max})$ and ϑ. This fixes L_q and the scale-harmonic grid $\{t_n\}$.
2. **Choose V_{APM}:** start with $V_{\mathrm{APM}} \equiv 0$ for calibration; then move to $V_{\mathrm{APM}}(q;\eta)$.
3. **Compute Hamiltonian band:** solve $H_{\mathrm{APM}}\psi_k = \lambda_k\psi_k$ up to $\lambda_k \le E_c$ (finite dimensional).
4. **Assemble S_{E_c}:** compute $(\mathsf{S}_{E_c})_{jk} = \langle\psi_j, W_I\psi_k\rangle_I$ using the kernel integral (11).
5. **Diagonalize and record:** compute eigenpairs of S_{E_c}; record $\Lambda_{\max}(E_c)$ and $\mathcal{J}(E_c)$ from (16)–(17).
6. **Scan parameters:** vary ϑ, L_q, and constitutive dials in V_{APM} to map coherent-surviving mode families and identify stable spectral features.

9 Standardized summary of Operator Package 3 (copy/paste block)

Operator Package 3 (Coherence-filtered spectral selection). Fix a finite log-scale window $I = [q_{\min}, q_{\max}]$ with $L_q = q_{\max} - q_{\min}$ and a holonomy phase ϑ. Let $\mathcal{H}_I = L^2(I)$. Define the discretized APM Hamiltonian $H_{\mathrm{APM}} = -\,\mathrm{d}^2/\,\mathrm{d}q^2 + V_{\mathrm{APM}}(q)$ with quasi-periodic boundary conditions $f(q_{\max}) = e^{\mathrm{i}\vartheta} f(q_{\min})$ and $f'(q_{\max}) = e^{\mathrm{i}\vartheta} f'(q_{\min})$. Let Φ be the theta-built xi-kernel from Operator Package 1 and define the compact self-adjoint filter $(W_I f)(q) = \int_I \Phi(q-u) f(u)\,\mathrm{d}u$. For an energy cutoff E_c, define the spectral projector $\Pi_{E_c} = \mathbf{1}_{(-\infty, E_c]}(H_{\mathrm{APM}})$ and the selection operator $\mathcal{S}_{E_c} := \Pi_{E_c} W_I \Pi_{E_c}$. Then $\mathcal{S}_{E_c}$ is finite-rank self-adjoint on $\mathrm{Ran}(\Pi_{E_c})$, with variational characterization $\Lambda_{\max}(E_c) = \max_{\|f\|=1,\ f \in \mathrm{Ran}(\Pi_{E_c})} \langle f, W_I f \rangle$. In the calibration case $V_{\mathrm{APM}} \equiv 0$, the diagonal matrix elements in the scale-harmonic basis are exact Fejér averages of Ξ: $\langle e_n, W_I e_n \rangle = \int \Xi(\tau) F_{L_q}(t_n - \tau)\,\mathrm{d}\tau$ with $F_{L_q}(x) = \frac{1}{2\pi} L_q \,\mathrm{sinc}^2(L_q x/2)$ and $\mathrm{sinc}(y) = \sin(y)/y$.

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