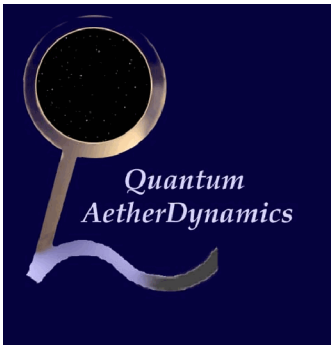

Quantum AetherDynamics Institute (QADI)

Operator Package 8: Canonical Determinant Closure (Module B) for the Riemann Function Operators Series

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Abstract

Module B is the analytic stabilization step in the Riemann Function Operators proof program: construct a canonically normalized limiting determinant $\Delta(t)$, obtained from trace-class Fredholm determinants of coherence-selected, windowed operators, and prove $\Delta(t) \equiv \Xi(t)$. This package promotes two core tools: (i) a determinant-topology double-limit convergence lemma for $(E \to \infty, L_q \to \infty)$, and (ii) an explicit canonical normalization rule $C(L_q, \vartheta; s, \lambda) = \Xi(0)/\Delta_{s,I}(0)$ together with a jet-continuity pipeline that reduces identity of entire functions to trace-class convergence of operator jets $K^{(k)}(0)$. A worked toy model (free-window Laplacian) and an explicit commutator-kernel lemma are included to make the jet mechanism concrete and repeatedly usable throughout the paper.

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1 Purpose and scope

Operator Package 5 isolates the determinant-closure bottleneck in the series: replace finite-rank determinants (unstable under rank growth) by a Fredholm determinant of a trace-class family, stable in trace norm and admitting convergent trace expansions [Thomson III, 2026e, Simon, 2005, Bornemann, 2010]. Operator Package 7 closes the Module A identification step by fixing the target multiplier $\Xi_{\mathrm{APM}} \equiv \Xi$ under the APM-derived theta partition [Thomson III, 2026g,d]. Module B now removes the cutoff $E \to \infty$ and the log-window size $L_q \to \infty$ and identifies the canonically normalized limit determinant with the classical Ξ function [Thomson III, 2026f, Titchmarsh, 1986].

Module B goal. Construct a canonically normalized limiting determinant $\Delta(t)$ satisfying

$$\Delta(t) \equiv \Xi(t) \qquad (t \in \mathbb{C}),$$

as entire functions. Positivity/canonical-system forcing of real zeros is deferred to Module C as organized in the proof-program fork [Thomson III, 2026e,f].

2 Standing definitions and hypotheses

2.1 Finite log-window Hilbert space and twist

Let $I = [q_-, q_+]$ with $L_q = q_+ - q_- > 0$, and let $\mathcal{H}_I := L^2(I, dq)$. Define the bounded multiplication operator $(Qf)(q) = qf(q)$ (bounded on I).

Let $\Phi_{\mathrm{APM}} : \mathbb{R} \to \mathbb{R}$ be real, even, and super-exponentially decaying, and define the windowed convolution operator

$$(W_I f)(q) := \int_{q_-}^{q_+} \Phi_{\mathrm{APM}}(q-u)\, f(u)\, du.$$

(See Operator Packages 3–4 for the APM kernel construction and coherence filtration on finite log-windows [Thomson III, 2026c,d].)

Define the unitary twist $(U_t f)(q) = e^{itq} f(q)$ and

$$W_I(t) := U_t W_I U_t^{-1}.$$

Its kernel on $I \times I$ is $K_t(q,u) = e^{it(q-u)} \Phi_{\mathrm{APM}}(q-u)$.

2.2 Trace-class renormalization

Let H_{APM} be a self-adjoint Hamiltonian on $\mathcal{H}_I$ (Sturm–Liouville on I with bounded potential and the same boundary holonomy as earlier packages [Thomson III, 2026a,b,c]). Fix $s > \frac{1}{2}$ and define

$$A_s := (I + H_{\mathrm{APM}})^{-s/2}.$$

Then $A_s \in \mathsf{S}_2$ on a one-dimensional finite window; this is the standard Sobolev weight mechanism for producing trace-class sandwiched operators [Simon, 2005, Reed and Simon, 1980]. Define the trace-class family

$$K_{s,I}(t) := A_s \, W_I(t) \, A_s \in \mathsf{S}_1.$$

Fix $\lambda \in \mathbb{R}$ and define the Fredholm determinant generator

$$\Delta_{s,I}(t) := \det\bigl(I + \lambda K_{s,I}(t)\bigr).$$

By trace-ideal analyticity, $\Delta_{s,I}(t)$ is even entire whenever Φ_{APM} is real and even [Simon, 2005].

2.3 Cutoff projections

Let $\Pi_E := \mathbf{1}_{(-\infty,E]}(H_{\mathrm{APM}})$ and define the finite-rank approximants

$$K_{s,I}^{(E)}(t) := A_s \, \Pi_E \, W_I(t) \, \Pi_E \, A_s, \qquad \Delta_{s,I}^{(E)}(t) := \det\bigl(I + \lambda K_{s,I}^{(E)}(t)\bigr).$$

This is the cutoff pathway introduced in the series to connect the operator family to finite-rank numerics and eigenvalue scans [Thomson III, 2026b,e].

3 Promoted item 1: determinant-topology double-limit convergence

3.1 A quantitative determinant continuity inequality

Lemma 3.1 (Determinant Lipschitz estimate in trace norm)**.** *Let $A, B \in \mathsf{S}_1$. Then*

$$\bigl|\det(I+A) - \det(I+B)\bigr| \;\le\; \|A-B\|_{\mathsf{S}_1} \exp\Bigl(1 + \|A\|_{\mathsf{S}_1} + \|B\|_{\mathsf{S}_1}\Bigr).$$

Proof. Standard in trace-ideal theory; see Simon [2005, Ch. 3–4]. Comparable perturbation bounds also appear in numerical Fredholm determinant discussions [Bornemann, 2010]. □

3.2 Cutoff removal at fixed window

Lemma 3.2 (Cutoff convergence in trace norm, uniform on compact t-sets)**.** *Fix a window I and $s > \frac{1}{2}$. Then $\|K_{s,I}^{(E)}(t) - K_{s,I}(t)\|_{\mathsf{S}_1} \to 0$ as $E \to \infty$, locally uniformly in $t \in \mathbb{C}$.*

Proof. Write

$$K_{s,I}(t) - K_{s,I}^{(E)}(t) = A_s(I - \Pi_E)W_I(t)A_s \; + \; A_s\Pi_E W_I(t)(I - \Pi_E)A_s.$$

Each term factors as $\mathsf{S}_2 \times \mathsf{S}_2$, hence is trace class [Simon, 2005]. Moreover, $\|A_s(I - \Pi_E)\|_{\mathsf{S}_2} \to 0$ by spectral decay of $(I + H_{\mathrm{APM}})^{-s/2}$ in one dimension, while $W_I(t)$ is bounded (and analytic) on a fixed window because Q is bounded on I [Reed and Simon, 1980, Simon, 2005]. □

Theorem 3.3 (Cutoff removal for determinants)**.** *For each fixed window I, $\Delta_{s,I}^{(E)}(t) \to \Delta_{s,I}(t)$ locally uniformly in $t \in \mathbb{C}$ as $E \to \infty$.*

Proof. Combine Theorem 3.2 with Theorem 3.1 [Simon, 2005]. □

3.3 Window removal and the double-limit lemma (with explicit scalar normalization)

To compare windows, use symmetric windows $I_L = [-L/2, L/2]$ of length L and write $\mathcal{H}_I^{(L)} := L^2(I_L)$. Let $K_s^{(L)}(t)$ denote the corresponding family.

Lemma 3.4 (Double-limit determinant convergence with explicit scalar normalization)**.** *Fix $s > \frac{1}{2}$ and $\lambda \in \mathbb{R}$. Assume:*

(i) (Cutoff limit) For each fixed L, $K_s^{(L,E)}(t) \to K_s^{(L)}(t)$ in S_1 locally uniformly in $t \in \mathbb{C}$ as $E \to \infty$.

(ii) (Window limit in S_1 after scalar normalization) There exists a scalar normalization $C(L) \neq 0$ and a S_1-valued entire limit family $K_s^{(\infty)}(t)$ such that

$$C(L)\, K_s^{(L)}(t) \to K_s^{(\infty)}(t) \quad \text{in } \mathsf{S}_1,$$

locally uniformly in $t \in \mathbb{C}$ as $L \to \infty$, and the family $\{C(L)K_s^{(L)}(t)\}$ is locally bounded in S_1 norm on compact t-sets.

Define

$$\Delta^{(L,E)}(t) := \det(I + \lambda K_s^{(L,E)}(t)), \qquad \Delta^{(\infty)}(t) := \det(I + \lambda K_s^{(\infty)}(t)).$$

Then the normalized double limit exists and is order-independent:

$$\lim_{L\to\infty} \lim_{E\to\infty} \det(I + \lambda C(L)K_s^{(L,E)}(t)) = \lim_{E\to\infty} \lim_{L\to\infty} \det(I + \lambda C(L)K_s^{(L,E)}(t)) = \Delta^{(\infty)}(t),$$

with local-uniform convergence in $t \in \mathbb{C}$.

Proof. Fix a compact $K \subset \mathbb{C}$. Use (ii) to choose L so that $\sup_{t\in K} \|C(L)K_s^{(L)}(t) - K_s^{(\infty)}(t)\|_{\mathsf{S}_1}$ is small; then for that L use (i) to choose E so that $\sup_{t\in K} \|C(L)K_s^{(L,E)}(t) - C(L)K_s^{(L)}(t)\|_{\mathsf{S}_1}$ is small. Then $\sup_{t\in K} \|C(L)K_s^{(L,E)}(t) - K_s^{(\infty)}(t)\|_{\mathsf{S}_1}$ is small. Apply Theorem 3.1 and local boundedness of trace norms on K [Simon, 2005]. □

4 Promoted item 2: canonical normalization and jet-continuity reduction

4.1 Canonical scalar normalization

Definition 4.1 (Canonical scalar normalization)**.** Assume $\Delta_{s,I}(0) \neq 0$ on the finite window I. Define the canonical scalar normalization by

$$C(L_q, \vartheta; s, \lambda) \;:=\; \frac{\Xi(0)}{\Delta_{s,I}(0)}.$$

Define the normalized determinant

$$\widetilde{\Delta_{s,I}}(t) := C(L_q, \vartheta; s, \lambda)\, \Delta_{s,I}(t).$$

Lemma 4.2 (Uniqueness among scalar normalizations)**.** *Let C be any scalar (depending on $L_q, \vartheta, s, \lambda$ but not on t) such that $C\,\Delta_{s,I}(0) = \Xi(0)$. Then $C = C(L_q, \vartheta; s, \lambda)$.*

Proof. Immediate. □

4.2 Operator jets: explicit commutator formulas

Lemma 4.3 (Iterated commutator jet formula)**.** *For each integer $k \geq 0$,*

$$\frac{d^k}{dt^k}\Big|_{t=0} W_I(t) = (\mathrm{i}\,\mathrm{ad}_Q)^k(W_I), \qquad \mathrm{ad}_Q(T) := [Q, T] = QT - TQ.$$

Consequently,

$$K_{s,I}^{(k)}(0) = A_s\,(\mathrm{i}\,\mathrm{ad}_Q)^k(W_I)\,A_s \in \mathsf{S}_1.$$

Proof. Since $U_t = e^{\mathrm{i}tQ}$ with bounded Q, $t \mapsto U_t T U_t^{-1}$ is norm-analytic for bounded T and $\frac{d}{dt} U_t T U_t^{-1} = \mathrm{i}\,U_t[Q, T]U_t^{-1}$ [Reed and Simon, 1980]. Iterate and evaluate at $t = 0$. The S_1 claim follows because $A_s \in \mathsf{S}_2$ and $(\mathrm{i}\,\mathrm{ad}_Q)^k(W_I)$ is bounded, so $A_s(\cdot)A_s$ is trace class [Simon, 2005]. □

4.3 Jet continuity pipeline

Lemma 4.4 (Cauchy formula for S_1-valued analytic maps)**.** *Let $F_n : \mathbb{C} \to \mathsf{S}_1$ be entire and suppose $F_n \to F$ locally uniformly in S_1 norm on $\mathbb{C}$. Then for every $k \geq 0$, $F_n^{(k)} \to F^{(k)}$ locally uniformly in S_1 norm.*

Proof. Cauchy's integral formula holds for Banach-space-valued holomorphic maps; see standard functional analysis references [Reed and Simon, 1980]. □

Lemma 4.5 (Determinant jets are continuous under S_1-jet convergence)**.** *Let $K_n : \mathbb{C} \to \mathsf{S}_1$ be entire and $K_n^{(j)}(0) \to K^{(j)}(0)$ in S_1 for $0 \leq j \leq m$. Define $\Delta_n(t) := \det(I + \lambda K_n(t))$ and $\Delta(t) := \det(I + \lambda K(t))$. Then for each $0 \leq k \leq m$,*

$$\Delta_n^{(k)}(0) \to \Delta^{(k)}(0).$$

Proof. The Fredholm determinant is Fréchet analytic on S_1, with bounded multilinear derivatives on bounded subsets [Simon, 2005]. Apply the finite-order chain rule expansion at $t = 0$. □

5 Purely internal closure: jet matching eliminates spurious prefactors

5.1 Even-entire identity by jets

Lemma 5.1 (Even-entire functions are determined by even jets)**.** *Let F and G be even entire functions. If*

$$F^{(2k)}(0) = G^{(2k)}(0) \quad \text{for all } k \geq 0,$$

then $F \equiv G$ on $\mathbb{C}$.

Proof. Evenness forces Maclaurin series in even powers only; equality of all even jets gives equality of coefficients. □

5.2 Moment jets for Ξ

A standard representation of Ξ as the Fourier transform of an even kernel yields

$$\Xi(t) = \int_{-\infty}^{\infty} \Phi_{\mathrm{APM}}(q)\, e^{\mathrm{i}tq}\, dq,$$

and therefore, by differentiation under the integral,

$$\Xi^{(2k)}(0) = (-1)^k \int_{-\infty}^{\infty} q^{2k}\, \Phi_{\mathrm{APM}}(q)\, dq,$$

with absolute convergence under super-exponential decay [Titchmarsh, 1986, Thomson III, 2026d].

Remark 5.2 (Why jets kill spurious prefactors)**.** If a window-removal limit produced $\Xi(t) \cdot e^{p(t)}$ where p is a nonzero even entire function, then some even jet at 0 must differ (e.g. e^{ct^2} forces $F^{(2)}(0)$ to shift by $2cF(0)$). Thus even-jet matching eliminates all nontrivial nonvanishing even entire prefactors automatically.

5.3 Jet-matching closure theorem

Theorem 5.3 (Jet-matching closure for Module B (fully internal))**.** *Assume:*

(A) (Module A fixed) $\Xi_{\mathrm{APM}} \equiv \Xi$ as entire functions [Thomson III, 2026g].

(B) (Canonical normalization) For each window I, define $\widetilde{\Delta_{s,I}}(t)$ by Theorem 4.1.

(C) (Window-removal existence) There exists an even entire limit $\Delta_{s,\infty}$ such that

$$\widetilde{\Delta_{s,I}}(t) \to \Delta_{s,\infty}(t) \quad \text{locally uniformly on } \mathbb{C} \text{ as } L_q \to \infty.$$

(D) (Even-jet matching) For every $k \geq 0$,

$$\Delta_{s,\infty}^{(2k)}(0) = \Xi^{(2k)}(0).$$

Then

$$\Delta_{s,\infty}(t) \equiv \Xi(t) \quad \text{on } \mathbb{C}.$$

Proof. Both $\Delta_{s,\infty}$ and Ξ are even entire. Apply Theorem 5.1. □

6 Discharging the remaining hypothesis (D): reduction to operator jets

Theorem 6.1 (Jet matching reduces to trace-class operator jets)**.** *Assume (A)–(C) of Theorem 5.3. Suppose additionally that there exists a* S_1*-valued entire limit family* $K_s^{(\infty)}(t)$ *such that, after applying the same scalar normalization as in Theorem 4.1 (absorbed into* $C(L)$*),*

$$C(L)\, K_s^{(L)}(t) \to K_s^{(\infty)}(t) \quad \text{locally uniformly in } \mathsf{S}_1 \text{ on } \mathbb{C}.$$

Then for every $k \geq 0$*,*

$$\widetilde{\Delta}_{s,I_L}^{(k)}(0) \to \Delta_{s,\infty}^{(k)}(0),$$

and $\Delta_{s,\infty}^{(k)}(0)$ *is a continuous functional of the operator jets* $\{K_s^{(\infty)\,(j)}(0)\}_{0\leq j\leq k}$ *in* S_1*.*

Proof. Local-uniform convergence in S_1 implies convergence of all operator jets by Theorem 4.4. Transfer jets through the determinant map using Theorem 4.5 [Simon, 2005]. □

7 Toy model: free window Laplacian (worked commutator-jet example)

7.1 Free Hamiltonian and Sobolev weight

Let $I = [-L/2, L/2]$ and take the free Hamiltonian

$$H_0 := -\frac{d^2}{dq^2}$$

with periodic boundary condition (holonomy $\vartheta = 0$), as a standard exactly-diagonalizable Sturm–Liouville reference model [Reed and Simon, 1980, Thomson III, 2026b]. Its eigenpairs are

$$e_n(q) = \frac{1}{\sqrt{L}} e^{\mathrm{i}(2\pi n/L)q}, \qquad H_0 e_n = \Big(\frac{2\pi n}{L}\Big)^2 e_n.$$

Then

$$A_s = (I + H_0)^{-s/2}$$

has singular values $\sigma_n = (1 + (2\pi n/L)^2)^{-s/2}$, hence $A_s \in \mathsf{S}_2$ for $s > \frac{1}{2}$ [Simon, 2005].

7.2 High-value lemma: commutator kernels are explicit and reusable

Lemma 7.1 (Commutator-kernel identity for windowed convolutions)**.** *Let W_I be the windowed convolution operator*

$$(W_I f)(q) = \int_I \Phi(q-u)\, f(u)\, du,$$

and let Q be multiplication by q on I. Then for each integer $k \geq 0$ the iterated commutator $(\mathrm{ad}_Q)^k(W_I)$ is an integral operator with kernel

$$(\mathrm{ad}_Q)^k(W_I) \quad \textit{has kernel} \quad (q-u)^k\, \Phi(q-u).$$

Consequently,

$$\frac{d^k}{dt^k}\Big|_{t=0} W_I(t) = \mathrm{i}^k\, (\mathrm{ad}_Q)^k(W_I) \quad \textit{has kernel} \quad \mathrm{i}^k (q-u)^k \Phi(q-u).$$

Proof. For $k = 1$,

$$([Q, W_I]f)(q) = q\int_I \Phi(q-u) f(u)\, du - \int_I \Phi(q-u)\, u f(u)\, du = \int_I (q-u)\Phi(q-u) f(u)\, du,$$

so the commutator kernel is $(q-u)\Phi(q-u)$. Induct: if $(\mathrm{ad}_Q)^k(W_I)$ has kernel $(q-u)^k\Phi(q-u)$, then applying ad_Q multiplies the kernel by another factor $(q-u)$. The twist-derivative identity follows from Theorem 4.3. □

7.3 Interpretation and repeated use

Theorem 7.1 is the concrete bridge between the analytic jet pipeline and numerical implementation: all operator jets at the origin reduce to sandwiched kernels

$$K_{s,I}^{(k)}(0) = A_s\Big(\mathrm{i}^k\, (\mathrm{ad}_Q)^k(W_I)\Big) A_s,$$

with $(\mathrm{ad}_Q)^k(W_I)$ explicitly given by $(q-u)^k\Phi(q-u)$. This identity is used repeatedly in Checkpoint B3 and in any attempted proof of the jet-matching hypothesis (D) via kernel moments, consistent with the operator-to-determinant reduction program [Thomson III, 2026e,f].

8 Order/type control from kernel decay (recorded explicitly)

Lemma 8.1 (Super-exponential kernel decay implies order ≤ 1)**.** *Assume Φ_{APM} is super-exponentially decaying and even. Then its Fourier transform*

$$\Xi(t) = \int_{\mathbb{R}} \Phi_{\mathrm{APM}}(q) e^{\mathrm{i}tq}\, dq$$

extends to an entire function of order at most 1 and finite exponential type.

Proof sketch. Super-exponential decay implies $\Phi_{\mathrm{APM}} \in L^1(\mathbb{R})$ with exponential weights, hence the Fourier transform is entire and satisfies bounds of the form $|\Xi(t)| \leq C \exp(\tau|\Im t|)$ for some $\tau > 0$, which yields order ≤ 1 and finite type; see standard treatments of Ξ and its integral representations [Titchmarsh, 1986]. □

9 Module B closure theorem (final form)

Theorem 9.1 (Module B closure: canonical limit + jet matching)**.** *Fix $s > \frac{1}{2}$ and $\lambda \in \mathbb{R}$. Assume:*

(1) Cutoff removal holds in S_1 locally uniformly in t (as in Theorem 3.2).
(2) The canonically normalized window-removal limit exists: $\widetilde{\Delta_{s,I}} \to \Delta_{s,\infty}$ locally uniformly in $t \in \mathbb{C}$ as $L_q \to \infty$.
(3) Even-jet matching holds at the origin: $\Delta_{s,\infty}^{(2k)}(0) = \Xi^{(2k)}(0)$ for all $k \geq 0$.

Then $\Delta_{s,\infty}(t) \equiv \Xi(t)$ on $\mathbb{C}$. Moreover, the determinant convergence is stable under the cutoff-window double limit in the sense of Theorem 3.4.

Proof. Identification is Theorem 5.3. Double-limit stability is Theorem 3.4. □

10 Operational checkpoints (falsifiable)

These checkpoints are the Module B diagnostic loop proposed in the proof program [Thomson III, 2026e,f].

10.1 Checkpoint B1: rank stability

Compute $\Delta_{s,I}^{(E)}(t)$ across increasing E and verify persistence of near-zeros under $E \to \infty$, then compare with $\Delta_{s,I}(t)$ via Theorem 3.3.

10.2 Checkpoint B2: window stability with canonical normalization

Enforce $\widetilde{\Delta_{s,I}}(0) = \Xi(0)$ via Theorem 4.1 and test stability of near-zeros under $L_q \to \infty$.

10.3 Checkpoint B3: jet stabilization diagnostics

Compute $\widetilde{\Delta_{s,I}}^{(2k)}(0)$ numerically (Cauchy differentiation) and compare against the moment values

$$\Xi^{(2k)}(0) = (-1)^k \int_{\mathbb{R}} q^{2k} \Phi_{\mathrm{APM}}(q)\, dq.$$

Agreement for increasing L_q is direct evidence for hypothesis (3) in Theorem 9.1.

A Determinants and trace ideals: minimal background

For $K \in \mathsf{S}_1$, the Fredholm determinant is

$$\det(I + K) = \prod_j (1 + \lambda_j),$$

and admits the absolutely convergent trace expansion

$$\log\det(I+K)=\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{m}\mathrm{Tr}(K^m),$$

with convergence controlled by $\|K\|_{\mathsf{S}_1}$ [Simon, 2005]. Numerical evaluation via Nyström quadrature is treated in Bornemann [2010].

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