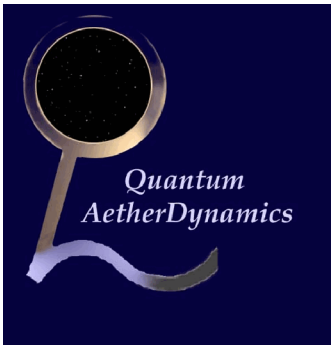

Quantum AetherDynamics Institute (QADI)

Final Consolidation: Identification, Determinant Closure, and Positivity Forcing for the Riemann Function Operators Program

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It develops the Aether Physics Model (APM)

Abstract

This note consolidates the Riemann Function Operators proof program into a single theorem chain. Module A identifies the APM-derived multiplier with the classical Riemann Ξ-multiplier. Module B states a canonical Fredholm-determinant closure criterion (cutoff removal, window-removal limit, and even-jet matching at the origin) implying the determinant limit equals Ξ as an entire function. Module C states a positivity certificate (Herglotz/de Branges forcing on growing windows) implying the determinant limit has only real zeros. Combining these steps yields the Riemann Hypothesis as a corollary under the explicit analytic hypotheses of Modules B and C.

1 Executive roadmap

The proof program is organized into three modules:

- **Module A (Identification).** The APM-derived Fourier multiplier Ξ_{APM} agrees identically with the classical Riemann Ξ on $\mathbb{C}$ [Thomson III, 2026a].
- **Module B (Determinant closure).** A canonically normalized, cutoff-removed, window-removal determinant limit $\Delta_{s,\infty}$ agrees identically with Ξ on $\mathbb{C}$ provided three explicit hypotheses hold [Thomson III, 2026b].
- **Module C (Positivity forcing).** If, on a sequence of growing windows, a determinant ratio is Herglotz and converges to a Herglotz limit, then the corresponding windowed determinants force the limit $\Delta_{s,\infty}$ to have only real zeros; hence Ξ has only real zeros and the Riemann Hypothesis follows [Thomson III, 2026c].

This consolidation paper makes the theorem chain explicit, isolates the remaining hypotheses, and packages the final corollary (RH) in a single place.

2 Classical Ξ and the RH equivalence

Let $\xi(s)$ denote the completed zeta function, and define the even entire function

$$\Xi(t) \;:=\; \xi\Big(\tfrac{1}{2} + \mathrm{i}t\Big), \qquad t \in \mathbb{C}.$$

It is standard that the Riemann Hypothesis is equivalent to:

All zeros of $\Xi(t)$ lie on the real axis in the t-plane.

(See Titchmarsh 1986.)

3 Standing Definitions (Packages 8–9 notation)

This section fixes the objects used throughout Modules B and C. Unless stated otherwise, all identities are in the setting of a bounded window and trace-class Fredholm determinants.

3.1 Windows, Hilbert space, and basic operators

Let

$$I = [q_-, q_+] \subset \mathbb{R}, \qquad L_q := q_+ - q_- > 0, \qquad H_I := L^2(I, dq).$$

Let Q be the bounded multiplication operator on H_I,

$$(Qf)(q) = q\, f(q).$$

3.2 Coherence kernel and windowed convolution

Fix a *coherence kernel* $\Phi_{\mathrm{APM}} : \mathbb{R} \to \mathbb{R}$ satisfying:

Φ_{APM} is real-valued, even, and rapidly (super-exponentially) decaying.

Define the windowed convolution operator $W_I : H_I \to H_I$ by

$$(W_I f)(q) = \int_{q_-}^{q_+} \Phi_{\mathrm{APM}}(q-u)\, f(u)\, du.$$

3.3 Twist operators and the twisted family $W_I(z)$

For $z \in \mathbb{C}$, define the unitary/analytic twist family by

$$(U_z f)(q) = e^{\mathrm{i}zq} f(q) = e^{\mathrm{i}zQ} f(q),$$

and the twisted family

$$W_I(z) := U_z\, W_I\, U_z^{-1}.$$

Equivalently, $W_I(z)$ is the integral operator on H_I with kernel

$$K_W(z; q, u) = e^{\mathrm{i}z(q-u)}\, \Phi_{\mathrm{APM}}(q-u), \qquad q, u \in I.$$

It satisfies the adjoint symmetry

$$W_I(z)^* = W_I(\overline{z}).$$

When $z = t \in \mathbb{R}$, we often write $W_I(t)$ for the real-frequency twist.

3.4 Sobolev sandwich and trace-class family $K_{s,I}(z)$

Fix a parameter $s > \frac{1}{2}$. Let H_{APM} denote the self-adjoint generator used in the series (the "APM Hamiltonian"), and define the bounded smoothing operator

$$A_s := (I + H_{\mathrm{APM}})^{-s/2}.$$

Define the trace-class family

$$K_{s,I}(z) := A_s\, W_I(z)\, A_s \in \mathcal{S}_1(H_I),$$

which inherits the adjoint symmetry

$$K_{s,I}(z)^* = K_{s,I}(\overline{z}).$$

3.5 Fredholm determinants and the ratio r_I

Fix $\lambda \in \mathbb{R}$. Define the two Fredholm determinants

$$\Delta^{+}_{s,I}(z) := \det\bigl(I + \lambda K_{s,I}(z)\bigr), \qquad \Delta^{-}_{s,I}(z) := \det\bigl(I - \lambda K_{s,I}(z)\bigr),$$

and the ratio (whenever $\Delta^{+}_{s,I}(z) \neq 0$)

$$r_I(z) := \frac{\Delta^{-}_{s,I}(z)}{\Delta^{+}_{s,I}(z)}.$$

For the Hermite–Biehler forcing package, define

$$E_I(z) := \Delta^{+}_{s,I}(z) - \mathrm{i}\,\Delta^{-}_{s,I}(z).$$

3.6 Cutoff projections and finite-rank approximants (Module B)

Let $\Pi_E := \mathbf{1}_{(-\infty,E]}(H_{\mathrm{APM}})$ be the spectral cutoff projection. Define the cutoff (finite-rank) trace-class operator

$$K^{(E)}_{s,I}(z) := A_s\,\Pi_E\,W_I(z)\,\Pi_E\,A_s,$$

and the corresponding cutoff determinant

$$\Delta^{(E),+}_{s,I}(z) := \det\bigl(I + \lambda K^{(E)}_{s,I}(z)\bigr).$$

(Analogously one can define $\Delta^{(E),-}_{s,I}$ with $-\lambda$.)

3.7 Canonical normalization (Module B)

Assuming $\Delta^{+}_{s,I}(0) \neq 0$, define the canonical scalar normalization

$$C(L_q, \vartheta; s, \lambda) := \frac{\Xi(0)}{\Delta^{+}_{s,I}(0)},$$

and the normalized determinant

$$\Delta^{g}_{s,I}(z) := C(L_q, \vartheta; s, \lambda)\,\Delta^{+}_{s,I}(z).$$

A *window-removal limit* is a locally uniform limit $\Delta^{g}_{s,I_L} \to \Delta_{s,\infty}$ on $\mathbb{C}$ along a sequence of growing windows I_L with $L_q \to \infty$.

3.8 Imaginary-part operators (Module C positivity bookkeeping)

For $z = a + \mathrm{i}b$ with $b > 0$, define the skew-adjoint part of $W_I(z)$ and its normalized form:

$$Y_W(z) := \frac{1}{2\mathrm{i}}\bigl(W_I(z) - W_I(z)^*\bigr) = \frac{1}{2\mathrm{i}}\bigl(W_I(z) - W_I(\overline{z})\bigr), \qquad H_W(z) := \frac{1}{\operatorname{Im} z}\,Y_W(z) = \frac{1}{b}\,Y_W(a + \mathrm{i}b).$$

Propagating through the Sobolev sandwich, define

$$Y_K(z) := \frac{1}{2\mathrm{i}}\bigl(K_{s,I}(z) - K_{s,I}(z)^*\bigr), \qquad H_K(z) := \frac{1}{\operatorname{Im} z}\,Y_K(z),$$

so that, by construction,

$$Y_K(z) = A_s\,Y_W(z)\,A_s, \qquad H_K(z) = A_s\,H_W(z)\,A_s.$$

3.9 Jet/commutator differentiation at the origin (Module B)

For real t, the twist satisfies $W_I(t) = U_t W_I U_t^{-1}$ with $U_t = e^{itQ}$. Hence

$$\frac{d^k}{dt^k}\Big|_{t=0} W_I(t) = (\mathrm{i}\,\mathrm{ad}_Q)^k(W_I), \qquad \mathrm{ad}_Q(T) := [Q,T] = QT - TQ,$$

and therefore

$$K_{s,I}^{(k)}(0) = A_s\,(\mathrm{i}\,\mathrm{ad}_Q)^k(W_I)\,A_s \in \mathcal{S}_1(H_I).$$

4 Module A: identification closure

We record the APM-derived construction in its minimal Fourier-kernel form, using the same kernel functional as Package 7.

Definition 4.1 (Kernel and multiplier construction)**.** Define the Jacobi theta series

$$\theta(x) := \sum_{n\in\mathbb{Z}} e^{-\pi n^2 x}, \qquad x > 0,$$

and set $\Theta(q) := \theta(e^{2q})$ for $q \in \mathbb{R}$. Define

$$G(q) := e^{q/2}(\Theta(q) - 1), \qquad \Phi_{\mathrm{APM}}(q) := \tfrac{1}{2}\Big(\tfrac{1}{4} - \partial_q^2\Big)G(q), \qquad \Xi(t) := \int_{-\infty}^{\infty} \Phi_{\mathrm{APM}}(q) e^{itq}\,dq.$$

Theorem 4.2 (Module A closure: $\Xi_{\mathrm{APM}} \equiv \Xi$ [Thomson III, 2026a])**.** *Under the canonical (unweighted) APM theta partition, the induced Fourier multiplier $\Xi_{\mathrm{APM}}(t)$ agrees with the classical Riemann multiplier $\Xi(t)$ as entire functions on $\mathbb{C}$.*

Remark 4.3. Module A removes the only ambiguity about the target function in the operator program: the determinant limit and positivity forcing are aimed at the classical Ξ.

5 Hypothesis Ledger (Modules B and C)

This section records, in one place, the explicit hypotheses required to pass from the finite-window trace-class determinants to the full Riemann Hypothesis via the consolidation theorem. All symbols and objects are as fixed in the Standing Definitions.

5.1 Module B hypotheses (canonical determinant closure)

Fix parameters $s > \frac{1}{2}$ and $\lambda \in \mathbb{R}$. Let $\{I_L\}$ be any sequence of windows with $I_L = [q_-(L), q_+(L)]$ and $L_q(L) = q_+(L) - q_-(L) \to \infty$.

(B1) Cutoff removal in trace norm (locally uniform in z). Let $\Pi_E = \mathbf{1}_{(-\infty,E]}(H_{\mathrm{APM}})$ and $K_{s,I}^{(E)}(z) = A_s \Pi_E W_I(z) \Pi_E A_s$. Assume that for every compact $K \subset \mathbb{C}$,

$$\sup_{z\in K} \big\| K_{s,I}^{(E)}(z) - K_{s,I}(z) \big\|_{\mathcal{S}_1} \xrightarrow[E\to\infty]{} 0,$$

uniformly for each fixed window I. Equivalently, cutoff-removed determinants satisfy

$$\Delta_{s,I}^{(E),+}(z) \to \Delta_{s,I}^{+}(z) \qquad \text{locally uniformly on } \mathbb{C}.$$

(B2) Window-removal limit for the canonically normalized determinants. Assume that the canonically normalized windowed determinants

$$\Delta_{s,I_L}^{g}(z) = \frac{\Xi(0)}{\Delta_{s,I_L}^{+}(0)}\,\Delta_{s,I_L}^{+}(z)$$

converge locally uniformly on $\mathbb{C}$ as $L \to \infty$ to an even entire function $\Delta_{s,\infty}$, i.e.

$$\Delta_{s,I_L}^{g} \longrightarrow \Delta_{s,\infty} \quad \text{locally uniformly on } \mathbb{C}.$$

(B3) Even-jet matching at the origin. Assume that for every integer $k \geq 0$,

$$\Delta_{s,\infty}^{(2k)}(0) = \Xi^{(2k)}(0).$$

Remark 5.1. Under **(B1)**–**(B3)**, Module B concludes the entire-function identity $\Delta_{s,\infty}(z) \equiv \Xi(z)$.

5.2 Module C hypotheses (positivity forcing / Herglotz certificate)

Fix the same parameters $s > \frac{1}{2}$ and $\lambda \in \mathbb{R}$. For each window I, define

$$\Delta_{s,I}^{\pm}(z) = \det\!\big(I \pm \lambda K_{s,I}(z)\big), \qquad r_I(z) = \frac{\Delta_{s,I}^{-}(z)}{\Delta_{s,I}^{+}(z)}, \qquad E_I(z) = \Delta_{s,I}^{+}(z) - \mathrm{i}\,\Delta_{s,I}^{-}(z).$$

(C1) Herglotz property on the upper half-plane (finite windows). Assume that for each L, the ratio r_{I_L} is Herglotz on $\mathbb{C}_+$; that is, r_{I_L} is analytic on $\mathbb{C}_+$ and satisfies

$$\operatorname{Im} r_{I_L}(z) \geq 0 \qquad (z \in \mathbb{C}_+).$$

(C2) Stability under window growth (normal family + Herglotz limit). Assume $\{r_{I_L}\}$ is locally bounded on $\mathbb{C}_+$ and converges locally uniformly on $\mathbb{C}_+$ to a limit r_∞, and that the limit remains Herglotz:

$$r_{I_L} \longrightarrow r_\infty \quad \text{locally uniformly on } \mathbb{C}_+, \qquad \operatorname{Im} r_\infty(z) \geq 0 \ (z \in \mathbb{C}_+).$$

(C3) Compatibility with the determinant limit. Assume that the "plus" determinants converge on $\mathbb{C}$ to the same limit function $\Delta_{s,\infty}$ appearing in Module B (after canonical normalization if required), in the sense that

$$\Delta_{s,I_L}^{+}(z) \;\longrightarrow\; \Delta_{s,\infty}(z) \quad \text{locally uniformly on } \mathbb{C},$$

up to the same scalar normalization already fixed in Δ_{s,I_L}^{g}.

Remark 5.2. Under **(C1)**–**(C3)**, Module C concludes that $\Delta_{s,\infty}$ has only real zeros. Combined with Module B ($\Delta_{s,\infty} \equiv \Xi$), this forces all zeros of Ξ to lie on $\mathbb{R}$, equivalently the Riemann Hypothesis.

6 Module B: canonical determinant closure

Theorem 6.1 (Module B closure [Thomson III, 2026b])**.** *Assume **(B1)–(B3)** from the Hypothesis Ledger. Then the window-removal limit $\Delta_{s,\infty}$ exists as an even entire function and satisfies*

$$\Delta_{s,\infty}(z) \equiv \Xi(z) \qquad (z \in \mathbb{C}).$$

7 Module C: positivity forcing

Theorem 7.1 (Module C closure [Thomson III, 2026c])**.** *Assume **(C1)–(C3)** from the Hypothesis Ledger and assume additionally that $\Delta_{s,\infty} \equiv \Xi$ (as delivered by Theorem 6.1). Then $\Delta_{s,\infty}$ has only real zeros. Consequently Ξ has only real zeros and the Riemann Hypothesis holds.*

8 Main consolidated theorem

Theorem 8.1 (Final consolidation: Modules A–C imply RH under explicit hypotheses)**.** *Assume:*

- *Module A identification closure holds (Theorem 4.2);*
- *Module B hypotheses (B1)–(B3) hold, yielding $\Delta_{s,\infty} \equiv \Xi$ (Theorem 6.1);*
- *Module C hypotheses (C1)–(C3) hold on a growing-window sequence, forcing real zeros for $\Delta_{s,\infty}$ (Theorem 7.1).*

Then the Riemann Hypothesis holds.

Proof. By Module A, the program's multiplier target coincides with the classical Ξ. By Theorem 6.1, hypotheses (B1)–(B3) imply $\Delta_{s,\infty} \equiv \Xi$. By Theorem 7.1, hypotheses (C1)–(C3) force $\Delta_{s,\infty}$ to have only real zeros, hence Ξ has only real zeros. Equivalently, all nontrivial zeros of $\zeta(s)$ lie on $\mathrm{Re}(s) = \frac{1}{2}$ [Titchmarsh, 1986]. □

9 Operational checkpoints (optional)

The proof program is designed so that each remaining hypothesis is falsifiable on finite windows: Nyström evaluation of determinants [Bornemann, 2010], trace-ideal control [Simon, 2005], and Herglotz/Pick-matrix tests for ratios [Thomson III, 2026c]. These checkpoints are not needed for the logical consolidation, but they map directly to the hypothesis ledger (B1)–(B3), (C1)–(C3).

Acknowledgments

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